St. Louis Community College Department of Mathematics &
St. Louis Community College Foundation

Present the 51st Annual

EXCELLENCE IN MATHEMATICS COMPETITION

St. Louis Area High School Mathematics Contest
November 4, 2023

With Special Thanks to

WOLFRAM   AND   TEXAS INSTRUMENTS

Prepared with help from the
American Mathematical Association of Two Year Colleges
and the Mathematics Department of
Rose-Hulman Institute, Terre Haute, Indiana

Instructions:
● You have 63 minutes to complete the items in this packet.
● Record all your responses on the answer sheet in spaces labeled # 1 through 20.
● Use the last page of this packet for scratch paper.
● No electronic or mechanical calculators are allowed.
● Each page has five items and every participant answers the same set of items.
● Each correct response is worth 5 points, no response earns 0 points, and every incorrect response earns -1 point.
● Do not mark your answer sheet during the first 60 minutes. Instead, write your responses down in this packet. You will receive clear instructions to mark your answer sheet with your final responses during the final three minutes. This will help you minimize erasures, which can affect your score negatively.
● Write all information in clear, block, capital letters.

Scoring Note: In the event of a tie score, item #20 is used as a tie-breaker. If ties still remain, item #19 will be used a tie-breaker, etc. A tie is resolved/won only with a correct response. Team ties are resolved based on the highest individual scorer for each team.

ITEM 1. Garlocks measure angles in hinglebits. A full circle measures 500 hinglebits. How many hinglebits are in a right angle?

A) 125   B) 112.5   C) 100   D) 90   E) none of these

ITEM 2. A glass is full of water. The total weight of the glass and water is 12 ounces. After drinking half the water, the weight of the glass and remaining water is seven ounces. What is the weight of the glass in ounces?

A) 1   B) 1.5   C) 2   D) 2.5   E) none of these

ITEM 3. If the operation $\bullet$ is defined so that $a \bullet b = ab(a + b)$, then $1\bullet(2\bullet3) =$

A) 162   B) 660   C) 900   D) 930   E) none of these

ITEM 4. A child wishes to color each of the six states on a map as shown. What is the minimum number of different color crayons she will need if each state must be a different color than all the states to which it is adjacent?


A) 2   B) 3   C) 4   D) 5   E) none of these

ITEM 5. A simplified form of $\dfrac{\dfrac{1}{a+b} - \dfrac{1}{a-b}}{2}$ is

A) $\dfrac{b}{b^2-a^2}$ B) $\dfrac{a}{a^2-b^2}$ C) $\dfrac{b}{a-b^2}$ D) $\dfrac{a}{b-a^2}$ E) none of these

ITEM 6. Fran and Zoey start at noon on a trip from P to Q and they travel at a constant speed, but not the same speed. If Fran takes eight hours for the trip and Zoey six hours, then how many hours after noon will Fran be twice as far from Q as Zoey?

A) 4.5   B) 4.8   C) 5   D) 5.2   E) none of these

ITEM 7. Two isosceles right triangles with side lengths 3 cm and 5 cm. are positioned as shown in the figure. The area of the shaded region (in cm^2) is
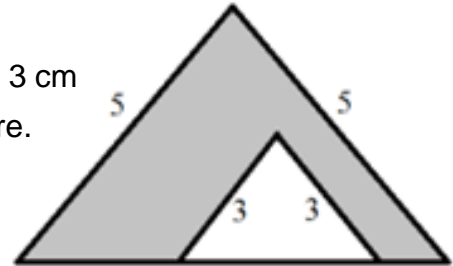
A) $2\frac{1}{2}$ B) 6   C) 8   D) 9   E) none of these

ITEM 8. A multiple choice test has five possible answers for each question. A correct answer is worth 5 points. An unanswered question is worth 0 points. How many points should be subtracted for an incorrect answer to eliminate any expected advantage or disadvantage of random guessing?
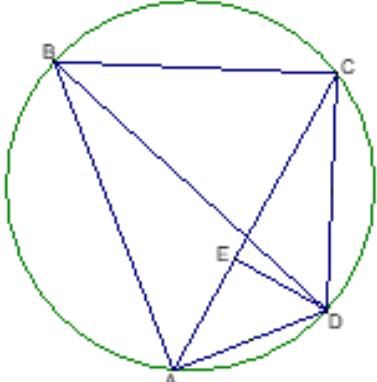
A) 1   B) 1.2   C) 1.25   D) 2   E) none of these

ITEM 9. The difference of two positive numbers is 8 and their product is 23. The sum of the two numbers is:

A) $1 + \frac{\sqrt{39}}{4}$ B) $\frac{-24+\sqrt{39}}{8}$ C) $\frac{4+\sqrt{39}}{16}$ D) $4\sqrt{39}$ E) none of these

ITEM 10. Replace each letter in ONE + ONE = TWO with a base-10 digit so that identical letters are replaced by identical digits and different letters are replaced with different digits, T is the only odd digit, and O cannot be zero. What is the value of N?

A) 2   B) 4   C) 6   D) 8   E) none of these

ITEM 11. Regular customers to a restaurant get a 10% discount, but then they pay a 5% tax and usually leave a 15% tip. The percentages are to be calculated in turn (with each percentage based on the previous calculation). Which order of the discounts/charge computations gives the lowest cost to the customer?

A) tax, tip, discount   B) tip, tax, discount   C) discount, tax, tip
D) discount, tip, tax   E) none of these

ITEM 12. If we call an integer "boring" if all its digits are the same, then how many integers greater than 1 and less than $10,000$ are both boring and prime?

A) 4   B) 5   C) 6   D) 7   E) none of these

ITEM 13. A grocery store sells oranges only in small bags of 4 oranges and large bags of 15 oranges (bags cannot be opened). Kara wants to purchase exactly N oranges but she cannot since she can only buy whole bags. However, she would be able to buy any amount greater than N (assuming an unlimited supply and endless funds). Determine N:

A) 14   B) 27   C) 39   D) 41   E) none of these

ITEM 14. Professor Ding-Dong likes to eat chocolates for an afternoon snack. On Monday morning he brings in a bag of 5 chocolates, 3 with red wrappers and 2 with green wrappers. At snack time every day, he reaches into the bag, pulls one out and eats it. What is the probability that the chocolate he eats on Friday will have a red wrapper?

A) $\dfrac{1}{5}$ B) $\dfrac{1}{3}$ C) $\dfrac{2}{5}$ D) $\dfrac{1}{2}$ E) none of these

ITEM 15. A $40\ yd.$ by $30\ yd.$ garden was subdivided into 1200 squares, each with side length $1\ yd.$ A post was placed at each corner of each square (only one post was placed on shared corners). A single section of fence of length $1\ yd$ was placed on each shared side and also along the outside border. Let P = the number of posts used and F = the number of fence sections used. Find $P + F$.

A) 3530   B) 3671   C) 3740   D) 3741   E) none of these

ITEM 16. Sara and David were reading the same novel. When Sara asked David what page he was reading, he replied that the product of the page number he was reading and the next page number was 100172. The sum of the digits for the page was David reading was

A) 13 B) 15 C) 17 D) 19 E) none of these

ITEM 17. Container A holds 60% acid solution and container B holds 80% acid solution. The two containers together hold a total of 300 liters of solutions. A tech removes x liters of solution from each container then adds the solution removed from container A to Container B, and vice versa. After thoroughly mixing the solutions in each container, container A holds 64% acid solution and container B holds a 78% acid solution. Find x:

A) 10 B) 20 C) 30 D) 40 E) none of these

ITEM 18. Suppose that the sum of the squares of two complex numbers w and z is 7, the sum of their cubes is 10, and their sum is a real number. What is the largest value $w + z$ can have?

A) 1 B) 2 C) 3 D) 4 E) none of these

ITEM 19. Quadrilateral $ABCD$ (shown, but not to scale) can be inscribed in a circle in such a way that BD is a diameter of the circle. Let E be the point on AC such that AC and DE are perpendicular. If $AE = 6$, $EC = 12$, and $DE = 5$, then the perpendicular distance from B to AC is

A) 13.3 B) 14.2 C) 14.4 D) 14.6 E) none of these

ITEM 20. You have a binary counter attached to a button. Each time you press the button, the counter increases by 1, expressing the number in binary notation. Each time a digit flips in the counter, you pay $\$1$. The counter is initially at zero. What is the cost of pressing the button 2023 times? (2023 in binary is 11111100111) For example, the cost of pressing the button 3 times is $\$4$, since it will go from 00 to 01 to 10 to 11 at the cost of $\$1 + \$2 + \$1$.

A) $\$2024$ B) $\$3995$ C) $\$4037$ D) $\$5161$ E) none of these