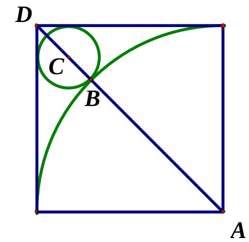
SOME SOLUTIONS

14. The least common multiple of 2 through 9 is $(2^3)(3^2)(5)(7) = 2520$, thus when any of the digits 2 through 9 is divided into 2521 the remainder will be 1.

15. Let R be the radius and A the center of the large circle. Let r *be* the radius and C the center of the small circle. Since $AD = R\sqrt{2}$ and $DC = r\sqrt{2}$, then $AD = AB + BC + DC = R + r + r\sqrt{2}$. Thus $r = \frac{R(\sqrt{2}-1)}{\sqrt{2}+1} = R(3-2\sqrt{2})$ and, for our problem, $R = 1/2$.



16. $v_S = c/t$, $v_J = b/t$. Hence $a = v_S + v_J = (c+b)/t$ and $a^2 = (c+b)^2/t^2$. Combining this expression with $a^2 = c^2 - b^2$ and $t = (c/b)^{3/2}$ gives

$$(c+b)^2 = (c/b)^3(c^2-b^2) \quad \Rightarrow \quad c+b = (c/b)^3(c-b) \quad \Rightarrow \quad (c/b)+1 = (c/b)^3((c/b)-1),$$

where we have divided both sides of the first equation by $c+b$ and both sides of the second by *b*.

Letting $x = c/b$ in the last equation, gives

$$x+1 = x^3(x-1) \;\Rightarrow\; (x^4-1) = x(x^2+1) \;\Rightarrow (x^2+1)(x^2-1) = x(x^2+1).$$

Dividing by x^2+1 and rearranging, gives $x^2 - x - 1 = 0$, and the positive solution of this quadratic is $x = (1+\sqrt{5})/2$. Thus $t = x^{3/2} = \left[\frac{1+\sqrt{5}}{2}\right]^{3/2}$.

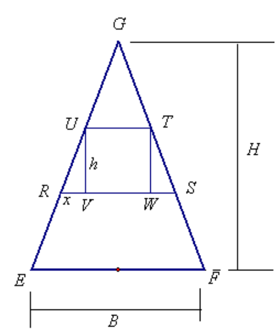
17. If b = number of blue, r = number of red, and x = the cost of the red, then $b + r = 17$, $b(x+1) + rx = 72$, Eliminating r gives $17x = 72 - b$ .Since x and b are positive integers with $b \le 17$, then $b = 4,\; r = 13$.

18. The length of the bases to the given altitudes are 2A/12, 2A/15, and 2A/20, where A is the area of the triangle. Thus the sides of the triangle are in proportion to 5,4,3 and the triangle is a right triangle whose area is half the product of the sides . Thus
A=(1/2) (2A/15) (2A/20), hence A=150 and the sides are 15 and 20 with hypotenuse 25. Thus perimeter=60. This can also be solved using Heron's formula.

19. Let *C*, *S*, *P* and *X* be the number of cows, sheep, pigs and chickens respectively, with dollar costs 10, 5, 2, and ½.. *X* must be even, since cost is an integer. If X = 100 or 98 or 96, then the chicken cost is \$50, \$49, \$48 and the remaining animals to total 100 can not bring the total cost to up \$100.

If *X* = 94, then the cost of cows, sheep and pigs must total \$53 and we must have an odd number of sheep, since the costs of cows and pigs are even. If *S* = 1, then there must be 5 animals from the cows and pigs at a cost of \$48. This can't happen since we would need at least 4 pigs to get the \$8. A similar argument holds for *S* odd and greater than 1. If *X* = 92 then a valid solution is $C = 4,\ S = 2, P = 2,\ X = 92$.

20. The figure shows one of the squares, *VWTU*, inscribed in the trapezoid *RSTU*. If *h* is the length of the square side and *x* the length of segment *RV*, then, by similar triangles, we have $(B/2)/H = x/h$. If *Q* is the ratio of the area of the square to the area of the trapezoid, then



$$Q = \frac{h^2}{h^2 + xh} = \frac{1}{1 + x/h} = \frac{1}{1 + \frac{B/2}{H}} = \frac{2H}{2H + B}.$$

Since the area of ΔEFG is equal to the sum of the areas of the sequence of trapezoids and the area of each square is *Q* times the area of its associated trapezoid, then the sum *S* of the areas of the sequence of squares is

$$S = Q\frac{BH}{2} = \frac{2H}{2H + B}\frac{BH}{2} = \frac{H^2 B}{2H + B}.$$

This result can also be found by summing the appropriate geometric series.