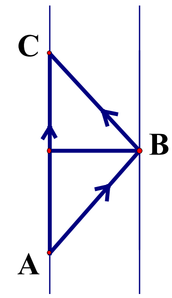
11. The sum of the digits of a two digit number is 10. If the digits are reversed then the new number exceeds the original number by 36. The product of the digits of the new number is

Solution: Let number be ab. 10a+b +36 = 10b+a also a+b = 10, solving : b = 7, a = 3. Thus the digit product is 21.

12. Zelma walks at 3 ft/sec along *AC* on the South side of a 30 ft wide street. Her husband Herb walks at 5 ft/sec from *A* to *B*, where *B* is on the North side and opposite the midpoint of *AC*. He then turns and walks to *C*. The figure is shown but not to scale. What is the length, in feet, of *AC* if they start together at *A* and arrive simultaneously at *C*?



Solution: Let D be the midpoint of AC. Let TZ be Zelma's travel time, TH be Herb's travel time and L the length of AC. TZ = L/3,

TH = $\dfrac{2\sqrt{30^2+(L/2)^2}}{5}$. Equating TH and TZ and solving gives L = 45.

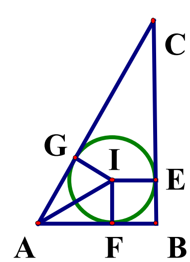
13. Given a rectangle R1 with area 5/2 and a second rectangle R2 with sides 1 greater than the corresponding sides of R1. If the area of R2 is numerically equal to the perimeter of R1 then the sum of their perimeters is

Solution: Let R1 have side lengths x and y and R2 have side lengths x+1 and y+1. Thus xy = 5/2 and $(x+1)(y+1) = 2x+2y$. Solving these equations gives the sides of R1 to be 1 and 5/2, the sides of R2 to be 2 and 7/2 and the perimeters sum to 18

14. The sum of the ages (integers) of Ann and Joe is 100. Joe is now three times as old as Ann was when Joe was 10 years older than Ann is now. The positive difference in their ages now is

Solution: let A and J be their ages now, also A-d and J-d be their earlier ages. Thus A+J = 100, J = 3(A-d) and J-d = A+10. Solving these three equations gives A = 37 and J = 63

15. If a circle is inscribed in a triangle with vertex angles 30°, 60°, and 90°, then the area of the triangle divided by the area of the circle is



Solution: Let *ABC* be the triangle as shown, *E, F,* and *G* the tangent points to the sides. The right triangles *AIF* and *AIG* are congruent and thus are both 30-60-90 triangles If r is the circle radius then $FB = r$, $AF = r\sqrt{3}$ and $AB = r(1+\sqrt{3})$. Also $BC = AB\sqrt{3} = r(3+\sqrt{3})$, since *ABC* is 30-60-90. Hence the area of *ABC* is $(AB)(BC)/2 = r^2(6+4\sqrt{3})/2$. The circle area is πr^2. Thus triangle area divided by circle area is $(3+2\sqrt{3})/\pi$

16. 16. On a single purchase you are given successive discounts of 5.5%, 10.5%, 14.5% and 12.5% in any order that you wish. Which discount should you choose first to get the largest total discount?

a) it does not matter b) 14.5 % c) 12.5% d) 5.5% e) none of these

Solution: Let $p_1 = 1 - .055,\ p_2 = 1 - .105, p_3 = 1 - .145, p_4 = 1 - .125$, then the final price is the initial price times the product of the respective p's in some order. Thus the ordering of the discounts does not change the final price.

17. Alice has a coin that comes up heads 60% of the time, otherwise tails. Bob's coin is 'fair', 50% heads and 50% tails. They each flip 2 times and Alice wins if she has more heads than Bob. Then the probability of Alice winning is

a) .32 b) .37 c) .39 d) .41 e) none of these

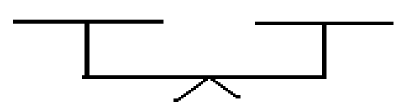
Solution: The table shows the probability of the various flips. Alice wins in the following events:

(1) Bob TT, Alice HH, HT, or TH Prob(1) = .25(.36+.24+.24) = .21
(2) Bob TH or HT, Alice HH Prob(2) = .5(.36) = .18

Result	Bob	Alice
HH	.25	.36
HT	.25	.24
TH	.25	.24
TT	.25	.16

Thus the probability of Alice winning is .21+.18 = .39.

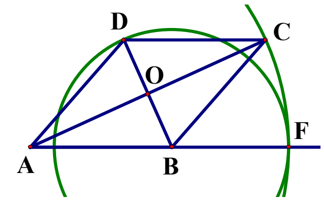
18. You are given 25 coins that look identical; however one of the coins is heavier than the others. You also have a pan balance. The minimum number of weighings needed to find the heavy coin is



Solution: Place 9 coins on left pan, 9 coins on the right pan and leave 7 coins on the table. If either pan goes down then the heavy coin is on the down pan. Now take the 9 that include the heavy and put three on left and three on right (after taking off the other 9). Now the heavy coin will be on the down pan or will be one of the remaining if the pans balance. In all three cases we have reduced the problem to finding the heavy coin that is known to be among three coins. Put one on left, one on right and the heavy is on the down pan. If they balance then the remaining coin is the heavy.

If the original 9 balance then the heavy is among the remaining 7. In this case you put 3 on the right pan and 3 on the left. If they balance the remaining coin is the heavy, otherwise the drill is as above. Thus the minimum number of weighings is 3.

19. A rhombus, ABCD, has sides of length 1. A circle with center A passes through C (the opposite vertex.) Likewise, a circle with center B passes through D. If the two circles are tangent to each other, then the length of BD is



Solution: For the rhombus $ABCD$ with center O, it is known or can be easily shown that triangles ABO, ADO, CDO and CBO are congruent right triangles. If $R = AC$ and $r = BD$ then $(R/2)^2 + (r/2)^2 = 1^2$ since AB has length 1 and is the hypotenuse of a triangle with side lengths $R/2$ and $r/2$. If F is the tangent point of the circles, then F is on the line through A and B since AF and BF are both perpendicular to the same tangent line through F. Thus $R = r + 1$, since AB has length 1.

Solving the eaquations $R^2 + r^2 = 4$ and $R = r + 1$ gives $r = \dfrac{-1 \pm \sqrt{7}}{2}$

20. A number is palindromic if it the same number when its digits are reversed, for example 14341. The odometer in Sally's car displays only 5 digits and does not display tenths of miles. When she started her trip, the last three digits formed a palindromic number. One mile later, the last four digits formed a palindromic number. After another mile, the last five formed a palindromic number. List all possible initial odometer readings that satisfy these conditions. The number of times the digit 9 appears in this list is

Solution: Let [a,b,c,d,c] be the initial digits of the odometer reading . Consider five cases and calculate the readings at miles 0,1,2, after imposing the palindromic requirment

Case I, c < 8 mile 0:[a,b,c,d,c] mile 1:[a,c+1,c,c,c+1] mile 2:[c+2,c+1,c,c,c+2].
Thus the last 4 digits at mile 2 cannot be a palindrome. Hence c = 8 or 9.

Case II, c = 8 mile 0:[a,b,8,d,8] mile 1:[a,9,8,8,9] mile 2:[0,9,8,9,0].
Required conditions satisfied for initial [0,9,8,8,8]

Case III, c = 9, d = 9, b=9 mile 0:[a,9,9,9,9] mile 1:[a+1,0,0,0,0] mile 2:[a+1,0,0,0,1].
Thus a = 0.
Required conditions satisfied for initial [0,9,9,9,9].

Case IV, c = 9, d = 9, b<9 mile 0:[a,b,9,9,9] mile 1:[a,b+1,0,0,0].
Since b+1 cannot be 0, then at mile 1 the last 4 digits cannot be a palindrome.

Case V, c = 9, d < 9 mile 0:[a,b,9,d,9] mile 1:[a,0,9,d+1,0] (thus d+1=9)
mile 2:[a,0,9,9,1] mile 2 cannot be a palindrome.

Thus there are two possible initial readings, 09888 and 09999 and there are five 9’s in these two odometer readings