SOLUTIONS, 2008, problems 11-20

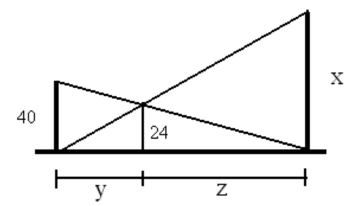
12. We are given $115 = 100A + 4B$ and $110 = 80A + 5B$. Solving the second equation for B in terms of A and substituting into the first gives $A = 3/4$ *and* $B = 10$. Thus
$P = (3/4)80 + (10)7 = 115$.

13. If the coordinates of C are (t, t^2), then the rectangle perimeter is $2t + 2t^2 = 24 \Rightarrow t = 3$. Thus $AC = \sqrt{9+81} = \sqrt{90}$, and the circle area = $90\pi/4$.

14. If the left pocket initially contained L dollars, then $L - L/4 - 20 = 100$. Thus $L = 160$ and the right pocket must have initially contained 40.

15. The only one of the numbers less than 1 is $\log_3 2$.

16. From similar triangles we have
$(y+z)/40 = z/24$ also $(y+z)/x = y/24$
Equating the two expressions for $(y+z)$ gives $z = xy/40$. Substituting this expression for z into the second equation gives $x = 60$.



17. If L is the length of the candles in inches, then $L / 6$ is the consumption rate of the slower candle in inches per hour and $F / 5$ is the faster rate. If t the time when slower is twice as tall as the faster, then $2(L - \frac{L}{5}t) = L - \frac{L}{6}t$. Thus $t = \frac{30}{7}$. .

18. Since $BN = BP$, $CP = CM$ and $AN = AM = 12$, then the perimeter of triangle ABC = 24. If BC has length x then the perimeter of the 30, 60, 90 triangle ABC is $x(1 + 2 + \sqrt{3}) = 24$, thus $x = 24/(3+\sqrt{3})$. If K is the area of ABC, then $K = \frac{1}{2}(x)(x\sqrt{3}) = \frac{24^2\sqrt{3}}{2(3+\sqrt{3})^2} = 48(2\sqrt{3} - 3)$.

19. If N is the integer whose digits are the first five digits of L, then $L = 10N + d$ and $M = 10^5 d + N$. Thus $10N + d = 3(10^5 d + N) \Rightarrow 7N = d(299999)$.
Hence for $d = 1$ we have $N = 42857$ and $L = 428571$, for $d = 2$ we have $N = 85714$ and $L = 857142$. For $d. > 2$, L has more than six digits and thus d is 1 or 2. The sum of the digits for these two possible solutions is 54.

20. Let $g(x) = f(x) - (2x - 1)$. Note that $g(x)$ is a polynomial of degree 4 with leading coefficient 1 and has zeros at $x = 1,\ 2$ and 3. Thus $g(x)$ can be factored as
$g(x) = f(x) - (2x-1) = (x-1)(x-2)(x-3)(x-k)$. Substituting $x = 0$ into this equation gives $f(0) + 1 = 6k$, and substituting $x = 4$ into this equation gives $f(4) - 7 = 6(4-k)$. Thus
$f(0) + f(4) = 30$.