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1. John, Mary and Sally make up a relay team. The team must run a mile and a half. If John runs 2/3 of a mile and Mary runs 3/4 of a mile, then how far must Sally run (in miles)?

a) 1/6 b) 1/4 c) 1/12 d) 1/8 e) none of these

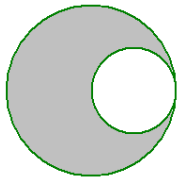
2. A simplified form of $\dfrac{\dfrac{1}{a+b}-\dfrac{1}{a-b}}{2}$ is

a) $\dfrac{b}{b^2-a^2}$ b) $\dfrac{a}{a^2-b^2}$ c) $\dfrac{b}{a^2-b^2}$ d) $\dfrac{a}{b^2-a^2}$ e) none of these

3. If $x+2y=4$ and $2x-3y=7$, then $x+y=$

a) 27/7 b) 26/7 c) 29/7 d) 25/7 e) none of these

4. The smaller of the two circles shown in the figure is tangent to the larger and passes through its center. If the larger radius is 3 then the area inside the larger and outside the smaller is



a) 9π b) $9\pi/2$ c) $9\pi-4\pi$ d) $3\pi/2$ e) none of these

5. The binary operation $\forall(x,y)$ is defined by $\forall(x,y)=x^2+x-y$. The value of $\forall(\frac{1}{2},\frac{1}{3})$ is

a) $\dfrac{1}{2}$ b) $\dfrac{3}{4}$ c) $\dfrac{7}{12}$ d) $\dfrac{5}{12}$ e) none of these

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6. A 2 gallon mixture of oil and gas contains 25% oil. If 1/2 gallon of oil is added to the original mixture, then the percentage of oil in the new mixture is

a) 30 b) 40 c) $33\frac{1}{3}$ d) 50 e) none of these

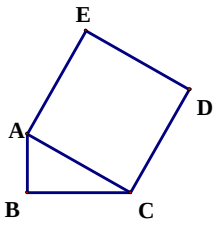
7. If 10 is the median of the list

$$n,\ n+3,\ n+4,\ n+5,\ n+6,\ n+8,\ n+10,\ n+12,\ n+15$$

then the average of the list is

a) 6 b) 8 c) 10 d) 11 e) none of these

8. *ACDE* is a square, *ABC* is a right angle, $BC=\sqrt{3}$, and $AB=\sqrt{2}$, as shown in the 'not to scale' figure. The area of the pentagon *BCDEA* is



a) $5+\sqrt{6}/2$ b) $4+\sqrt{5}$ c) $5+\sqrt{6}$ d) $4+\sqrt{6}/2$ e) none of these

9. Eight miles of fence are used to enclose a square plot. A second plot is enclosed with 15 miles of fence. The second plot is rectangular with the length twice the width. The sum of the two plot areas, in square miles, is

a) 20.5 b) 18.6 c) 16.5 d) 13.5 e) none of these

10. The union of any sets *A* and *B* is denoted by $A\cup B$. Their intersection is denoted by $A\cap B$. For any two sets A and B the set $(A\cap B)\cup A$ can be expressed as

a) A b) B c) $A\cup B$ d) $A\cap B$ e) none of these

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11. How many integers *x* are there such that $1 \le x \le 100$ and $x^3 + 3x + 1$ is divisible by 5? Note that *a* is divisible by *b* means that if *a* is divided by *b*, then the remainder is 0.

a) 20 b) 24 c) 40 d) 44 e) none of these

12. In the cryptogram the letters represent distinct digits. There are no carries in the addition, thus **O, N**, and **E** represent digits that are all less than 5 and equal to or greater than 0. How many different addition problems could this cryptogram represent?

$$\begin{array}{r} \text{O N E} \\ +\text{O N E} \\ \hline \text{T W O} \end{array}$$

a) 7 b) 6 c) 2 d) 4 e) none of these

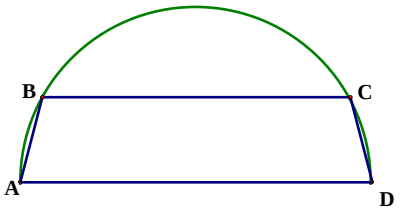
13. Find the area in the first quadrant bounded by the line $x = 1$ on the left, the line $x = 10$ on the right, the line $y = 0$ below, and the function $y = [[x]^{1/2}]$ above. The bracket function is used twice and denotes the greatest integer function, for example $[3.72] = 3$. Note that $a^{1/2} = \sqrt{a}$.

a) 13 b) 14 c) 15 d) 16 e) none of these

14. The absolute value of *x* is denoted by $|x|$. The product of all real values of *x* satisfying the equation $2|x|^2 + |x| = 6$ is

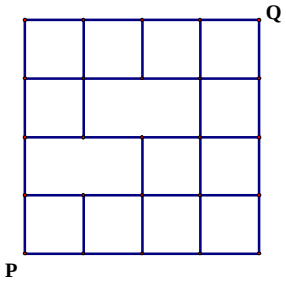
a) 9/4 b) -9/4 c) -9 d) 9 e) none of these

15. A trapezoid *ABCD* is inscribed in a semicircle of radius 2. They share the same base *AD*. If the lengths of segments *AB* and *DC* are both 1, then the area of *ABCD* is



a) $16/\sqrt{17}$ b) $16\sqrt{15}/15$ c) $\sqrt{17}$ d) $\sqrt{15}$ e) none of these

16. How many distinct paths are there from *P* to *Q* if you must stay on the line segments and at each intersection you must go up or to the right?

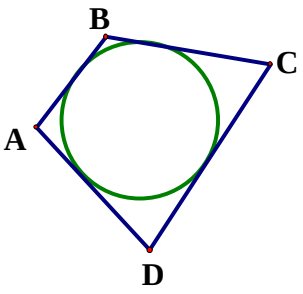


a) 24 b) 38 c) 32 d) 28 e) none of these

17. Given a sphere of radius 1 inscribed in a cube (tangent to all 6 faces). A smaller sphere is nestled in the corner such that it is tangent to the given sphere and also tangent to three intersecting faces of the cube. The radius of the smaller sphere, to two decimals, is

a) 0.17 b) 0.24 c) 0.14 d) 0.27 e) none of these

18. A circle is inscribed in the quadrilateral *ABCD* as shown (not to scale). The sides *AB*, *BC*, and *CD* have lengths 8, 5, and 10 respectively. The length of side *DA* is



a) 11 b) 9 c) 13 d) 10 e) none of these

19. Three Jeeps, J_1, J_2, J_3, are at base camp on the edge of a desert with full tanks of gas. They are to help J_1 drive to an oasis deep into the desert with J_2 and J_3 returning safely back to base camp and J_1 remaining at the oasis. J_1 and J_2 travel together for a spell and then J_2 refills J_1. J_1 then continues to the oasis and J_2 turns around to head back towards base until empty where it is met by J_3. J_3 has stayed at base camp, to later head out and share gas with the returning J_2, so that they can both get back to base. If the tanks hold 30 gallons and the Jeeps get 20 miles per gallon, then the maximum distance, in miles, that the oasis can be from base camp is

a) 900 b) 2600/3 c) 2800/3 d) 2900/3 e) none of these

20. Let *ABCD* be a regular tetrahedron with each of its six sides of length 1. Let *E* be any point in face *ABC*. Let *s* the sum of the distances from *E* to the faces *DAB*, *DBC* and *DCA*, and *S* be the sum of distances from *E* to the edges *AB*, *BC* and *CA*. Then S/s is

a) $3/(2\sqrt{2})$ b) $3\sqrt{3}/4$ c) $2/\sqrt{3}$ d) $\sqrt{3}/\sqrt{2}$ e) none of these